

Universality in nonequilibrium condensation of exciton-polaritons

Michał Matuszewski and Emilia Witkowska

Instytut Fizyki Polskiej Akademii Nauk, Aleja Lotników 32/46, 02-668 Warsaw, Poland

We investigate the process of condensation of exciton-polaritons in a one-dimensional nanowire, predicting spontaneous formation of domains of uncondensed excitons and condensed polaritons. We demonstrate a universal scaling law for the density of domains, which results from the competition between characteristic timescales present in the system. However, we find that the system does not follow the standard Kibble-Żurek scenario of a nonequilibrium phase transition.

PACS numbers: 67.85.De, 71.36.+c, 03.75.Kk

Semiconductor microcavities, in which photons are confined and strongly coupled to excitons, allow for creation of exciton-polaritons. These bosonic quasi-particles can undergo Bose-Einstein condensation at standard cryogenic or even room temperature [1–4]. The achievement of exciton-polariton condensate opened a path for new technological developments, eg. in optoelectronics (ultra fast switches, quantum circuits) [5] or medicine (compact terahertz lasers) [6]. It is known that condensation can have a character of a nonequilibrium phase transition when it takes place on a finite timescale [8]. Such transitions have been attracting great attention in many fields of science [7]. One of the early studies concerned topological defect formation in cosmological models [9]. As the hot universe cooled down, the phase transitions broke symmetries of the vacuum fields, which led to the creation of topological defects. This mechanism has been also considered in a wide variety of other physical models, including superfluid Helium [10], superconductors, cold atomic gases, and other systems [11]. The outcome of the transition is the creation of defects such as domain walls, monopoles, strings, vortices or solitons [8–11]. Bose-Einstein condensates of exciton polaritons, as highly controllable nonequilibrium quantum systems, are very suitable for studying these phenomena.

The Kibble-Żurek mechanism (KZM) theory is a powerful tool that allows to predict the density of defects without solving the full dynamical equations. It is assumed that the system adiabatically follows the equilibrium state up to the “freeze-out” time \hat{t} before the critical point. At this moment the relaxation time becomes too long for the system to adjust to the change of parameters. Consequently, the fluctuations approximately freeze, and the system enters the “impulse” phase. After crossing the critical point, the adiabaticity is restored when the relaxation time becomes short enough again. If the system in the final phase exhibits symmetry breaking, distant parts will choose to break the symmetry in different ways. The number of resulting defects will be determined by the “memorized” healing length $\hat{\xi}$ at the time \hat{t} . Remarkably, the time \hat{t} and the number of defects N_d can be determined solely from the critical exponents

of the phase transition, according to the scaling laws

$$|\hat{t}| \sim v^{-z\nu/(1+z\nu)}, \quad N_d \sim \hat{\xi}^{-d} \sim v^{d\nu/(1+z\nu)}, \quad (1)$$

where v is the rate of parameter change, d is the dimensionality, z and ν are the critical exponents. All the details of complicated dynamics are hidden, but we are still able to derive Eqs. (1) thanks to the underlying universality. The Kibble-Żurek scaling laws have been confirmed numerically in many cases, including the condensation of ultracold atoms [8].

The condensation of exciton-polaritons, on the other hand, is an example of a qualitatively different case. This phase transition may be named “nonequilibrium” in yet another meaning of this word. In fact, there may not be a well defined equilibrium state that the system could follow, due to the lossy nature of the system itself [7]. The particle loss and gain may be so strong that they to a large extent determine the relaxation of the system.

A natural question arises, whether the Kibble-Żurek scenario does apply to the case of an intrinsically nonequilibrium system such as the polariton condensate? And do the critical scaling laws such as the one given by Eq. (1) hold? In this letter, we aim to answer these questions by investigating the exciton-polariton domain formation during condensation in a one-dimensional GaAs nanowire [3]. We note that recently it was suggested that the KZM describes the formation of quantum vortices in a related two-dimensional system [4]. However, no evidence of the above adiabatic-impulse-adiabatic scenario or the critical scalings was given. We show that in the case considered here the system does not follow this scenario, and consequently the dynamics cannot be adequately described by the standard KZM. Nevertheless, power laws similar to Eq. (1), originating from the competition of characteristic timescales in the system, are still present.

We model polariton condensation in an incoherently pumped nanowire within the truncated Wigner approximation adapted to the polariton case [12]. The stochastic equations for the polariton order parameter $\psi(x, t)$ and the reservoir exciton density $n_R(x, t)$ are (in the quasi-

one-dimensional version)

$$d\psi = -\frac{i}{\hbar}dt \left[-\frac{\hbar^2 \nabla^2}{2m^*} + U(x) + i\frac{\hbar}{2} (R^{1D}n_R - \gamma_C) + (g_C^{1D}|\psi|^2 + g_R^{1D}n_R) \right] \psi + dW, \quad (2)$$

$$\frac{\partial n_R}{\partial t} = P(x, t) - \gamma_R n_R - R^{1D}n_R|\psi|^2,$$

where m^* is the effective mass of lower polaritons, $U(x)$ is the disorder potential due to imperfections of the sample, R is the rate of stimulated scattering, γ_C and γ_R are the polariton and exciton loss rates, g_C and g_R are the respective interaction coefficients, $P(x, t)$ is the external pump profile, and dW is a complex stochastic variable where

$$\langle dW(x)dW(x') \rangle = 0, \quad (3)$$

$$\langle dW(x)dW^*(x') \rangle = \frac{dt}{2\Delta x} (R^{1D}n_R + \gamma_C) \delta_{x,x'},$$

reflecting the quantum noise due to the particles entering and leaving the condensate. The coefficients R , g_C , g_R are renormalized in the quasi-one-dimensional case by assuming a Gaussian perpendicular profile of $|\psi|^2$ and n_R of width d determined by the nanowire thickness, $(R^{1D}, g_i^{1D}) = (R, g_i)/\sqrt{2\pi d^2}$.

We consider the simplest case of a flat pump beam of extent L , switched on at the time $t = 0$, $P(x, t) = P_0 \theta(t) \theta(L/2 - |x|)$, with $\theta(x)$ being Heaviside step function. Nevertheless, the general conclusions should hold for other spatial and temporal pulse shapes, as well as electrical pumping, as long as the pump is switched on rapidly. The above model describes the dynamics of the system at zero temperature. In reality, the distribution of excited modes may resemble a thermal state with temperature comparable to the ambient temperature [1, 13]. However, such mode distribution should not influence the phenomena presented here. We will see that the correlation length in the condensed phase is equal to several tens of micrometers, order of magnitude larger than the correlation (de Broglie) length in the thermal state at a typical temperature of 10 K. Therefore, the excited mode distribution can be well approximated by a flat distribution in momentum space, in accordance with Eqs. (3).

In Fig. 1 we present results of a typical simulation of the condensation process. The Figures 1(a) and (b) show the polariton and reservoir exciton density in a single simulation, which can be interpreted as a single experimental realization (pulse). The polariton condensate appears after about 350 ps after switching the pump on, in the form of areas of high density, complemented by areas of high incoherent exciton density. While the polariton domains appear to be vulnerable and erratic structures (dissolving after few hundreds of picoseconds), they become much more pronounced and well defined when averaged over

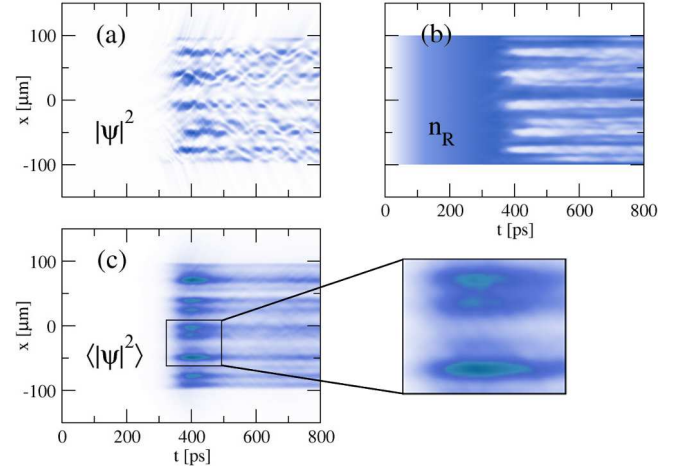


FIG. 1. Evolution of (a) polariton density $|\psi|^2$ and (b) exciton reservoir density n_R during condensation in a 1D wire with a disorder potential of amplitude $U_0 = 0.05$ meV using a top hat beam with $P_0 = 5P_{th}$. (c) Polariton density averaged over 100 pulses. The magnified image shows three of the visible polariton domains created due to the nonequilibrium nature of the phase transition. Other parameters are $\gamma_C = 1/30$ ps, $\gamma_R = 1/400$ ps, $d = 5\mu\text{m}$, $R = 2.8 \times 10^{-3} \mu\text{m}^2 \text{ps}^{-1}$, $g_C = 3\mu\text{eV} \mu\text{m}^2$, $g_R = 2g_C$.

many realizations of the stochastic equations, which corresponds to averaging over many pulses, see Fig. 1(c). Here we included a small realistic disorder potential $U(x)$ with amplitude of fraction of meV and correlation length of a few micrometers. It allows to pin the domains in certain regions of the sample, that would otherwise become smeared out over many realizations. The apparent exciton-polariton phase separation originates from the repulsion between the two components. It is related to the existence of two stationary solutions to the homogeneous version of Eqs. (2). Substituting $\psi(x, t) = \psi_0 e^{-i\mu t}$ and $n_R(x, t) = n_{R0}$ we get

$$\begin{aligned} (a) \quad & |\psi_0|^2 = 0, & n_{R0} &= \frac{P_0}{\gamma_R}, \\ (b) \quad & |\psi_0|^2 = \eta \frac{\gamma_R}{R^{1D}}, & n_{R0} &= \frac{\gamma_C}{R^{1D}}, \end{aligned} \quad (4)$$

where $\eta = P_0/P_{th} - 1$ and $P_{th} = \gamma_C \gamma_R / R^{1D}$ is the threshold value of the pump power. These are the exciton and polariton phases, the latter existing only for $P_0 > P_{th}$. Above P_{th} the system may choose locally to approach one of them.

Let us turn our attention to the initial dynamics of the system. Initially, both the polariton and exciton modes are almost empty. After switching the pump on, the reservoir exciton density n_R in the illuminated area starts to increase gradually. The polariton field, on the other hand, is damped or amplified depending on the value of $\varepsilon(x, t) = R^{1D}n_R - \gamma_C$, see Eq. (2). The condition $\varepsilon = 0$ determines the critical point, and ε has the same role as the relative temperature in the case of a thermal phase

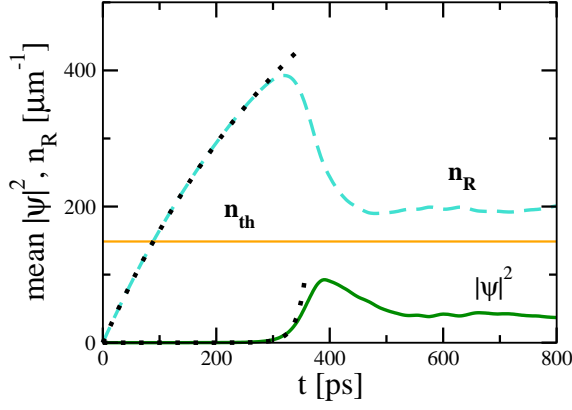


FIG. 2. Evolution of the mean polariton density $|\psi|^2$ and reservoir density n_R in Fig. 1 averaged over the illuminated area. The vertical line corresponds to the condensation threshold of n_R when the polariton component becomes unstable. The dotted lines correspond to the analytical results (5)-(6).

transition [10]. We are not able to control ε directly, but by varying the pump power P_0 , we can control the rate of the phase transition by changing the slope of $\varepsilon(t)$ at the critical point.

The initial evolution of polariton and exciton occupations can be well approximated with an analytical solution which neglects the nonlinear terms in Eqs. (2). For convenience, we now choose $t = 0$ at the time where the reservoir exciton density reaches the polariton amplification threshold $n_R = n_{th} = R^{1D}/\gamma_C$, and obtain

$$n_R(x, t) = \frac{\gamma_C}{R} \left(\frac{P_0}{P_{th}} - \eta e^{-\gamma_R t} \right), \quad (5)$$

$$N(t) = N(0) \exp \left[\frac{\gamma_C}{\gamma_R} \eta (\gamma_R t + e^{-\gamma_R t} - 1) \right], \quad (6)$$

where $N(t) = \int |\psi(x, t)|^2 dx$ is the number of condensed polaritons. Figure 2 shows excellent agreement of this simplified model with numerics up to the instant when the polariton density saturates due to the nonlinearity.

The crucial observation is that while polariton density $|\psi|^2$ remains small, the relaxation is completely determined by the gain and damping terms, and the related timescale is given by $t_{rel} = 1/|\varepsilon|$. One of the consequences of strong nonequilibrium nature of the system is that there is practically no mixing of modes with different momentum, and consequently the relaxation does not establish any relevant length scale (healing length), as $\varepsilon = \gamma_C \eta (1 - e^{-\gamma_R t})$ is independent of momentum. Contrary to the KZM, there is no well defined equilibrium before the phase transition, and the mode occupation is mainly shaped by the noise correlations (3). Moreover, as we show below, it is not these initial correlations that determine the final number of created defects.

The situation changes when the polariton-polariton interaction becomes sufficiently strong to compete with

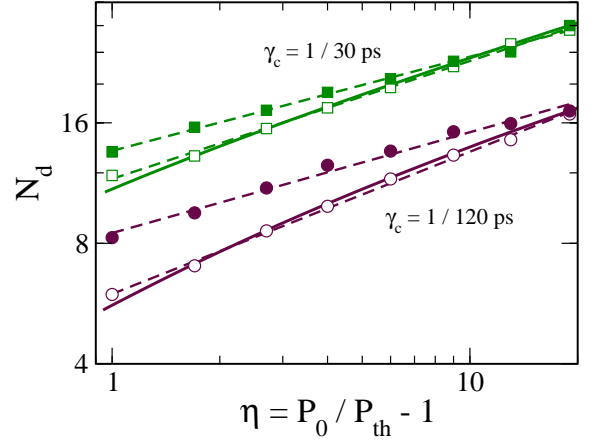


FIG. 3. Average number of polariton domains as a function of the relative power in bilogarithmic scale. Full points correspond to experiments averaged over 30 different samples with $U_0 = 0.05$ meV, and empty points correspond to a sample without disorder. For each sample, 50 Wigner realizations were done. The solid line is the solution of the analytical model Eqs. (5)-(8). The scaling exponents from linear fits (dashed lines) in the case $\gamma_C = 1/30$ ps are 0.24 (with disorder) and 0.29 (no disorder), and the respective exponents in the case $\gamma_C = 1/120$ ps are 0.25 and 0.36.

nonequilibrium processes

$$\varepsilon(\tilde{t}) = \alpha g_C^{1D} \bar{n}(\tilde{t}), \quad (7)$$

with α being a constant of the order of unity, and $\bar{n}(t) = N(t)/L$ the average polariton density in the illuminated area. At this point \tilde{t} , which can be identified as the condensation time [13], mode mixing starts to take place, which results in the appearance of an instantaneous quasi-equilibrium. It is this moment, when the length scale associated with the size of emerging domains is established. The number of created domains is determined by the interaction-dependent healing length $\xi = (\hbar^2/2m^* g_C^{1D} n)^{1/2}$ at the time \tilde{t} ,

$$N_d \sim L/\xi \sim (g_C^{1D} \bar{n}(\tilde{t}))^{1/2}, \quad (8)$$

with L being the system size.

In Figure 3 we show the dependence of the average number of created domains [14] on the pumping parameter η . In the absence of disorder, the agreement between the numerical data and the theoretical model of Eqs. (5)-(8), depicted by solid lines, is very good over the power range of $P_0 = 2P_{th} \dots 20P_{th}$ for two different values of γ_C . This confirms the validity of our scenario of domain formation, based on the transition from the strongly nonequilibrium to the quasi-equilibrium regime. We note that the depicted dependence is well approximated by power-law functions (straight dashed lines) both in the presence and absence of the small disorder potential. In the presence of disorder, the slope is somewhat smaller,

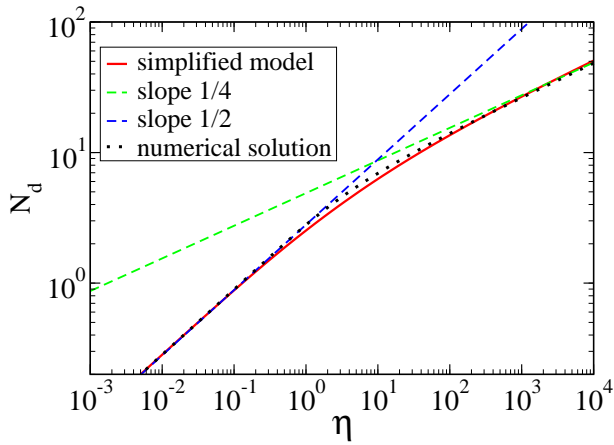


FIG. 4. Theoretical predictions for the number of domains N_d as a function of η . Solid line corresponds to the analytical approximation (9), dotted line is the numerical solution of Eqs. (5)-(8), green dashed line corresponds to the limit $\eta \gg \gamma_R/\gamma_C$, and blue dashed line to the limit $\eta \ll \gamma_R/\gamma_C$. Parameters are the same as in Fig. 3 with $\gamma_R = 1/30$ ps.

which we contribute to the ability of disorder wells and hills to shape additional domains. We found out that even in the case of very small P_0 , there was a minimum number of domains induced by the presence of the disorder.

A simplified analytical solution can be obtained by expanding Eq. (6) up to the second order in $\gamma_R t$. Doing this, one deals with third-order equation for the condensation time \tilde{t} which has the physical (real and positive) solution

$$2\gamma_R \tilde{t} = 1 + \frac{1}{D} + D, \quad (9)$$

where $D = (a - 1 + \sqrt{a(a - 2)})^{1/3}$ and $a = 12C\gamma_R/\eta\gamma_C$, with C logarithmically dependent on the parameters. The resulting scaling of domain number is presented in Fig. 4. In the limit $\eta \gg \gamma_R/\gamma_C$ the number of domains scales like $N_d \sim \eta^{1/4}$, while in the limit $\eta \ll \gamma_R/\gamma_C$ the scaling is $N_d \sim \eta^{1/2}$. The transition between the two regimes is determined roughly by $\eta \approx \gamma_R/\gamma_C$, and takes place over few orders of magnitude of η . Therefore, for a limited range of η , the scaling law may still appear as power-law with exponent between 1/2 and 1/4, in agreement with the results of Fig. 3.

Finally, we comment on the relation of our scenario of domain formation in an intrinsically nonequilibrium system with the standard Kibble-Zurek mechanism. It is clear that some of the key elements of the KZ mechanism, such as the adiabatic-impulse-adiabatic scenario, or the “freezing” of correlation length before the phase transition, are missing in the polariton condensation scenario. Nevertheless, in both cases, the scaling laws result from the competition of timescales existing in the system. Consequently, they will have similar mathemat-

ical forms. If we assume that the control parameter ε changes like $\varepsilon \sim (vt)^\kappa$, the local departure of the order parameter from the initial “vacuum” state is proportional to $|\psi|^2 \sim \exp(v^\kappa t^{\kappa+1})$, and the quasi-equilibrium is achieved at $\tilde{t} \sim v^{-\kappa/(1+\kappa)}$, up to a logarithmic factor. The correlation length at this point is $\xi \sim v^{-\kappa/z(1+\kappa)}$. These formulas are reminiscent of the standard KZ scalings (1). For example, in the $\eta \gg 1$ limit $\varepsilon(t) \sim t$ is a good approximation over the entire dynamics up to \tilde{t} , and we have $\kappa = 1$ and $z = 2$ before condensation, which results in the correct prediction $N_d \sim \eta^{1/4}$.

In conclusion, we investigated the formation of coherence of exciton-polariton condensate in a one-dimensional nanowire. We demonstrated the spontaneous formation of exciton and polariton domains and found that the standard adiabatic-impulse-adiabatic scenario of the Kibble-Zurek theory cannot describe the system dynamics. Nevertheless, power-law critical scalings for the number of created defects and the condensation time are still present, which results from the competition between characteristic timescales in the system. Our study can have practical implications for applications proposed for polariton systems, as we point to the possible difficulty in creating a coherent state on a finite timescale.

We thank Jacek Dziarmaga for useful discussions. This work was supported by the National Science Center grants DEC-2011/01/D/ST3/00482, DEC-2011/03/D/ST2/01938, and by the Foundation for Polish Science through the “Homing Plus” program.

-
- [1] J. Kasprzak et al., *Nature (London)* **443**, 409 (2006).
 - [2] S. Christopoulos et al., *Phys. Rev. Lett.* **98**, 126405 (2007); M. Wouters and I. Carusotto, *Phys. Rev. Lett.* **99**, 140402 (2007); A. Amo, J. Lefrère, S. Pigeon, C. Adrados, C. Ciuti, I. Carusotto, R. Houdré, E. Giacobino, and A. Bramati, *Nat. Phys.* **5**, 805 (2009); G. Nardin, K. G. Lagoudakis, M. Wouters, M. Richard, A. Baas, R. André, L. S. Dang, B. Pietka, and B. Deveaud-Plédran, *Phys. Rev. Lett.* **103**, 256402 (2009); H. Deng, H. Haug, and Y. Yamamoto, *Rev. Mod. Phys.* **82**, 1489 (2010).
 - [3] E. Wertz et al., *Nat. Phys.* **6**, 860 (2010).
 - [4] K. G. Lagoudakis, F. Manni, B. Pietka, M. Wouters, T. C. H. Liew, V. Savona, A. V. Kavokin, R. André, and B. Deveaud-Plédran, *Phys. Rev. Lett.* **106**, 115301 (2011).
 - [5] T. C. H. Liew, A. V. Kavokin, and I. A. Shelykh, *Phys. Rev. Lett.* **101**, 016402 (2008); I. A. Shelykh, A. V. Kavokin, Y. G. Rubo, T. C. H. Liew, and G. Malpuech, *Semicond. Sci. Technol.* **25**, 013001 (2010); C. Adrados, T. C. H. Liew, A. Amo, M. D. Martín, D. Sanvitto, C. Antón, E. Giacobino, A. Kavokin, A. Bramati, and L. Viña, *Phys. Rev. Lett.* **107**, 146402 (2011); D. Ballarín et al., arXiv:1201.4071 (2012).
 - [6] A. V. Kavokin, I. A. Shelykh, T. Taylor, and M. M. Glazov, *Phys. Rev. Lett.* **108**, 197401 (2012).
 - [7] G. Nicolis and I. Prigogine, *Self-organization in nonequi-*

- librium systems: from dissipative structures to order through fluctuations* (Wiley, 1977); H. Haken, *Synergetics: An Introduction : Nonequilibrium Phase Transitions and Self-Organization in Physics, Chemistry and Biology* (Springer, Berlin, 1978); H. Hinrichsen, *Physica A* **369**, 1 (2006).
- [8] B. Damski and W. H. Żurek, *Phys. Rev. Lett.* **104**, 160404 (2010).
- [9] T. W. B. Kibble, *J. Phys. A* **9**, 1387 (1976); T. W. B. Kibble, *Phys. Rep.* **67**, 183 (1980).
- [10] W. H. Żurek, *Nature (London)* **317**, 505 (1985); W. H. Żurek, *Phys. Rep.* **276**, 177 (1996); C. Bauerle et al., *Nature (London)* **382**, 332 (1996); V.M.H. Ruutu et al., *ibid.* **382**, 334 (1996).
- [11] I. Chuang et al., *Science* **251**, 1336 (1991); A. Maniv, E. Polturak, and G. Koren, *Phys. Rev. Lett.* **91**, 197001 (2003); L. E. Sadler, J. M. Higbie, S. R. Leslie, M. Vengalattore, and D. M. Stamper-Kurn, *Nature (London)* **443**, 312 (2006); C. N. Weiler et al., *Nature (London)* **455**, 948 (2008); E. Witkowska, P. Deuar, M. Gajda, and K. Rzażewski, *Phys. Rev. Lett.* **106**, 135301 (2011).
- [12] I. Carusotto and C. Ciuti, *Phys. Rev. B* **72**, 125335 (2005); M. Wouters and V. Savona, *Phys. Rev. B* **79**, 165302 (2009); D. Read, T. C. H. Liew, Y. G. Rubo, and A. V. Kavokin, *Phys. Rev. B* **80**, 195309 (2009).
- [13] A. V. Kavokin, J. J. Baumberg, G. Malpuech, and F. P. Laussy, *Microcavities* (Oxford University Press, Oxford, 2007).
- [14] To determine the number of domains N_d we count the number of zero crossings of the function $f(x) = n(x) - 0.5n_{\max}$, where n_{\max} is the maximum of local polariton density over the entire experiment, at the time instant when the number of these crossings is the largest. We checked that this method gives reliable estimation of N_d .